Lessons 030 - 032

Wednesday, November 22

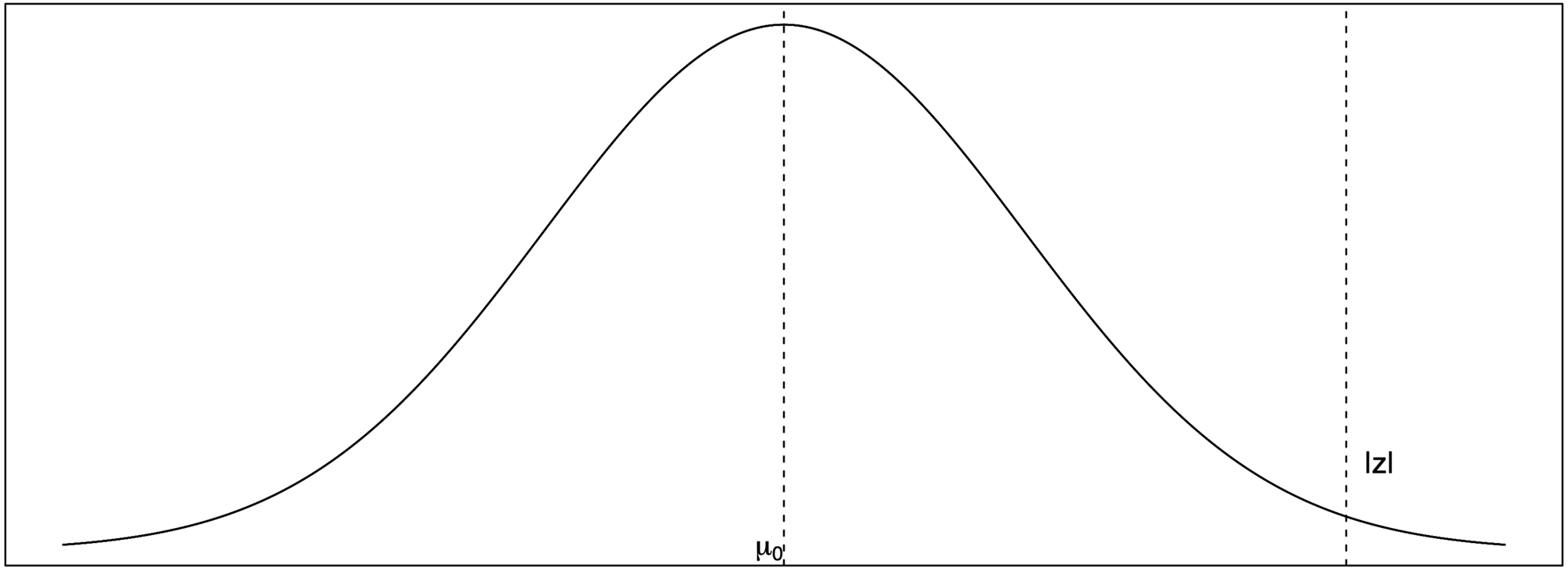
The Z Test for Population Means

- Suppose we have a sample from a normal population, with known variance (σ) and unknown mean.
- We wish to test: $H_0 : \mu = \mu_0$ versus the alternative, $H_1 : \mu \neq \mu_0$.
  - May also consider $H_0 : \mu \leq \mu_0$ versus $H_1 : \mu > \mu_0$ or $H_0 : \mu \geq \mu_0$ versus $H_1 : \mu < \mu_0$.
- We have see the sampling distribution for

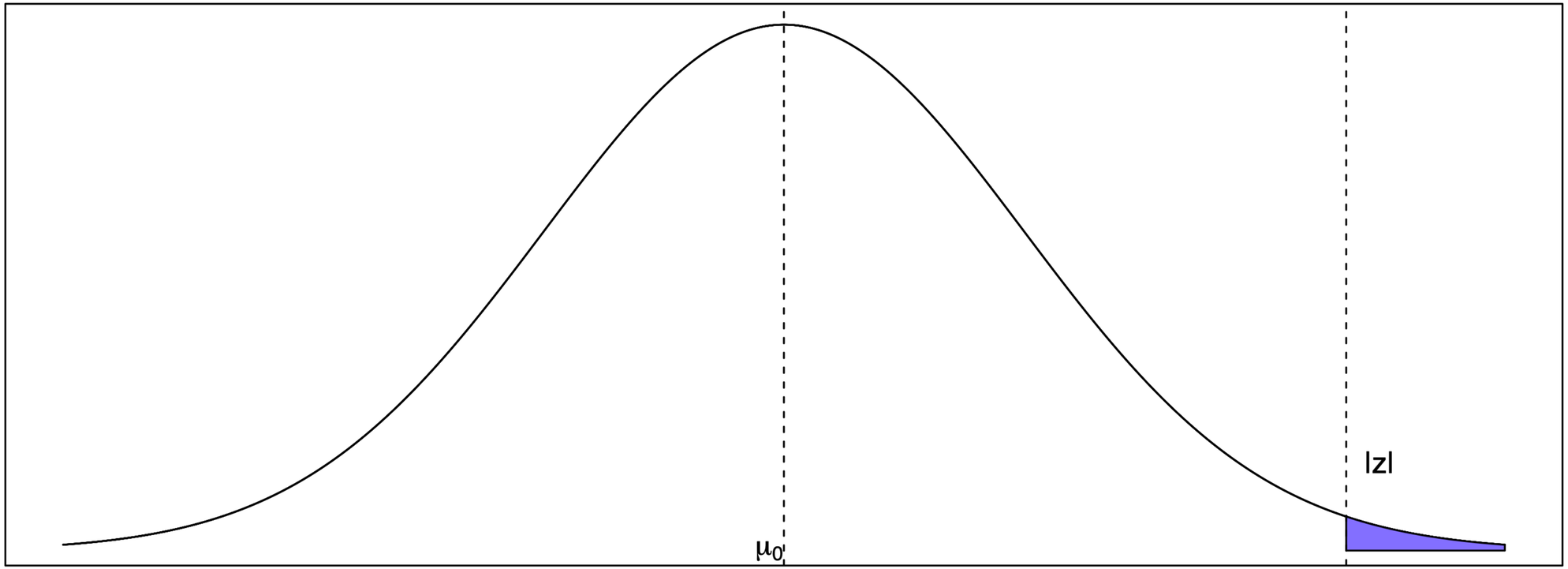
$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

  - If μ_0 is the correct mean, this will be $N(0, 1)$.
  - Note: we do **not** need a normal population for this.
- How do we find our p-value?

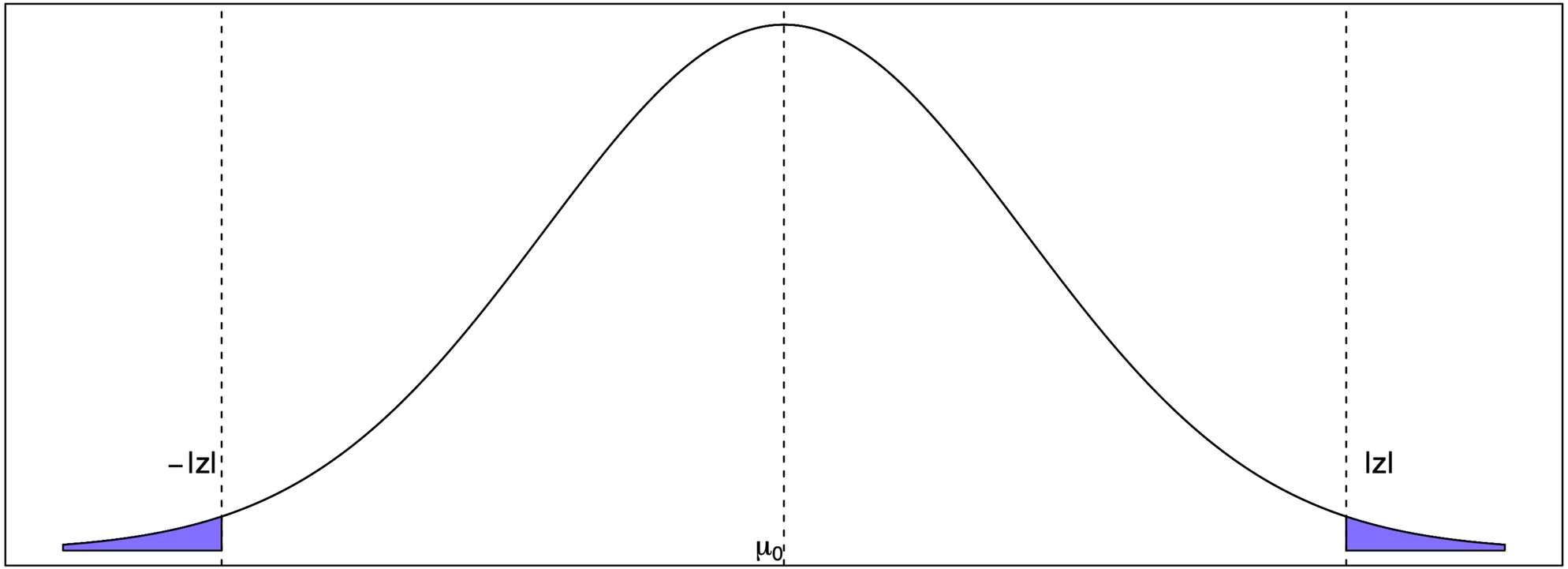
Calculating p-values



Calculating p-values



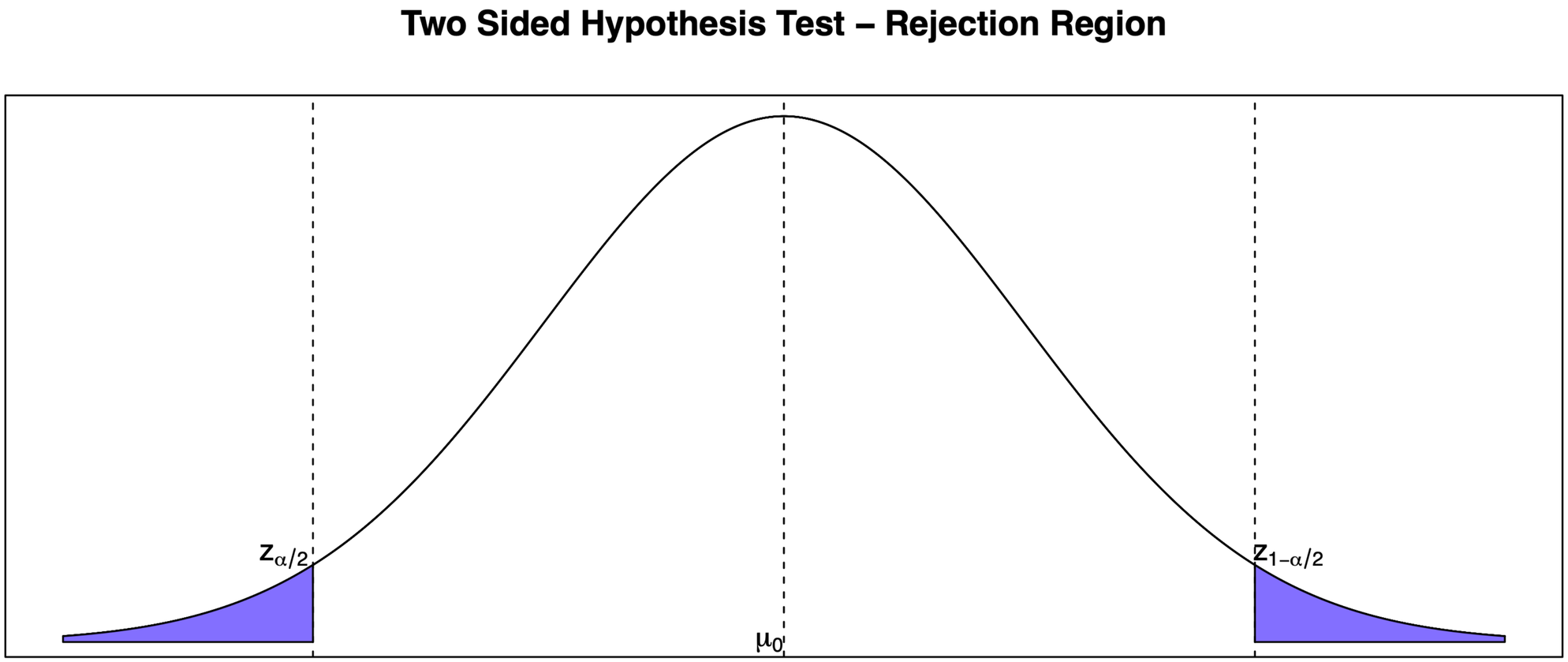
Calculating p-values



P-Value for (Two Sided) Tests

- If we observe z, we want to compute $P(|Z| \geq |z|)$.
- For symmetric distributions this is given by $P(Z \geq |z|) + P(Z \leq -|z|)$.
  - Can use the tables, critical values, or statistical software we have seen before.
- If our null hypothesis is one-sided then one of the two tail regions does *not* provide evidence against H_0.
  - If $H_0 : \mu \geq \mu_0$, then only consider $P(Z \leq z)$.
  - If $H_0 : \mu \leq \mu_0$, then only consider $P(Z \geq z)$.
  - Note here we do not take the absolute value.

Rejection Regions for Hypothesis Tests - Critical Values



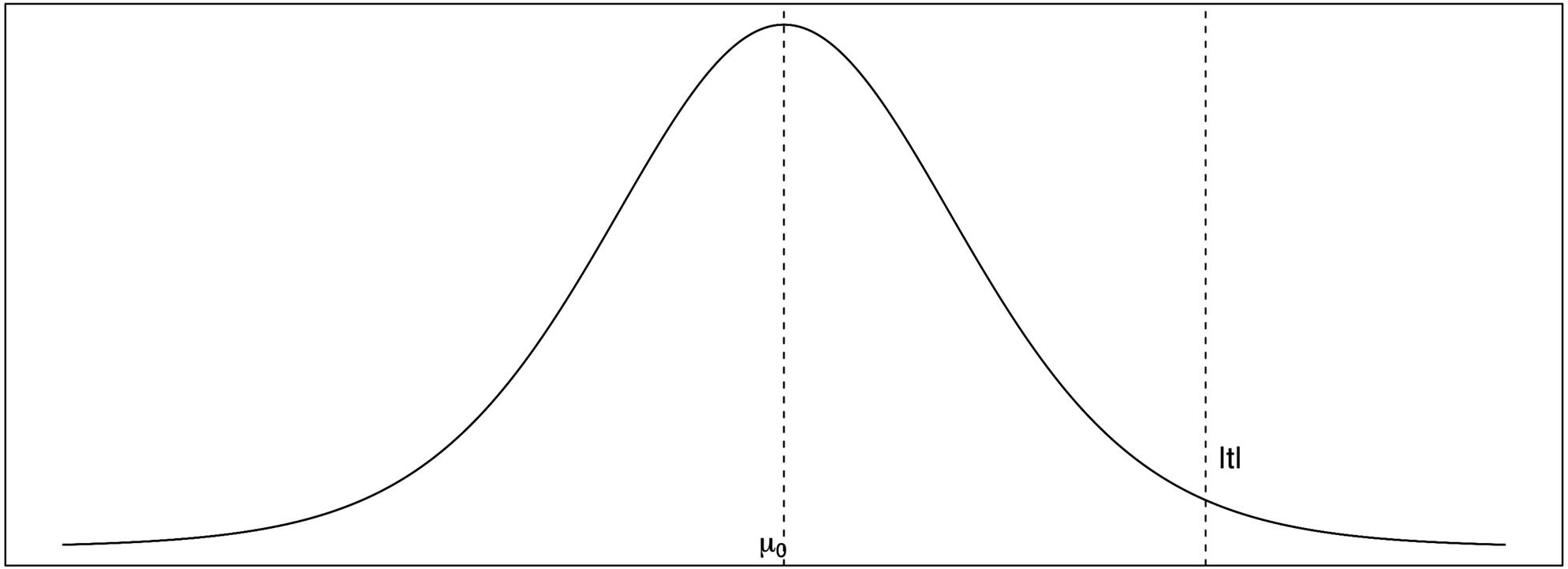
The One Sample t Test

- Suppose we have a sample from a normal population, with unknown variance and unknown mean.
- We wish to test: $H_0 : \mu = \mu_0$ versus the alternative, $H_1 : \mu \neq \mu_0$.
  - May also consider $H_0 : \mu \leq \mu_0$ versus $H_1 : \mu > \mu_0$.
  - May also consider $H_0 : \mu \geq \mu_0$ versus $H_1 : \mu < \mu_0$.
- We have see the sampling distribution for

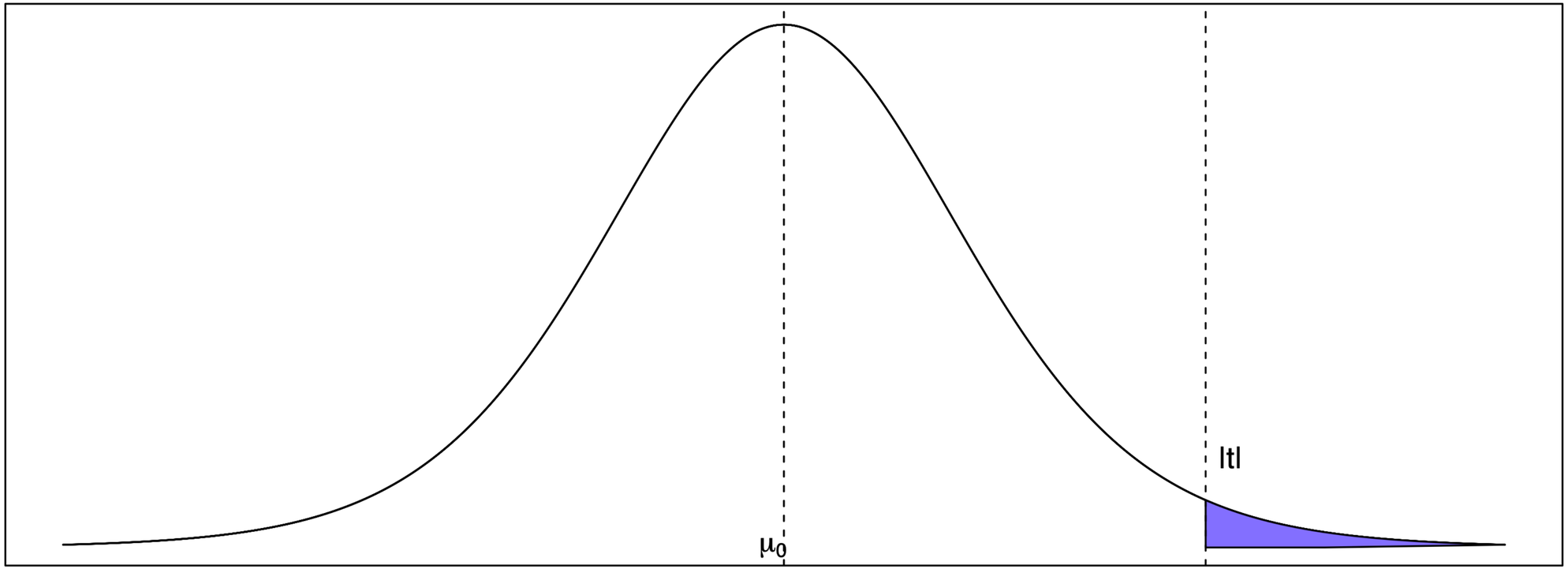
$$T = \frac{\overline{X} - \mu_0}{s/\sqrt{n}}.$$

  - If μ_0 is the correct mean, this will be t_{n-1}.
- Finding the p-value is equivalent to the $N(0, 1)$ case, substituting the normal distribution for a t distribution.

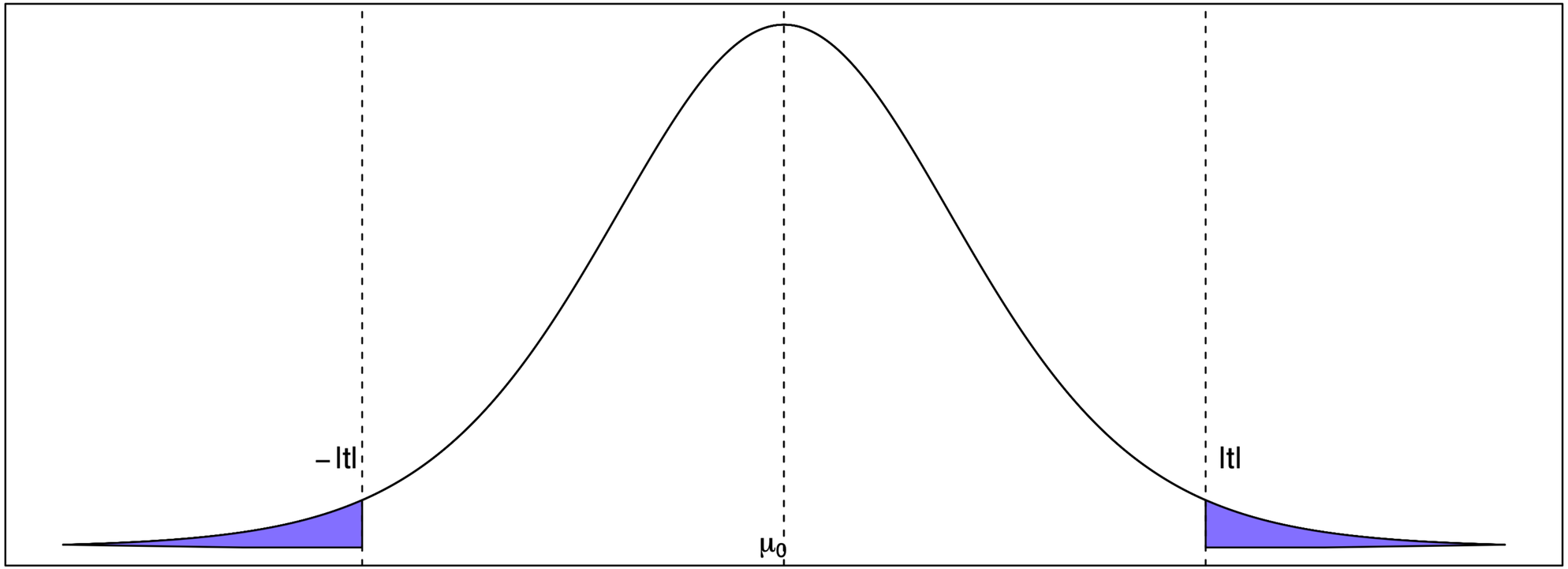
Calculating p-values



Calculating p-values



Calculating p-values



P-Value for (Two Sided) Tests

- If we observe t, we want to compute $P(|T| \geq |t|)$.
- For symmetric distributions this is given by $P(T \geq |t|) + P(T \leq -|t|)$.
- If our null hypothesis is one-sided then one of the two tail regions does *not* provide evidence against H_0.
  - If $H_0 : \mu \geq \mu_0$, then only consider $P(T \leq t)$.
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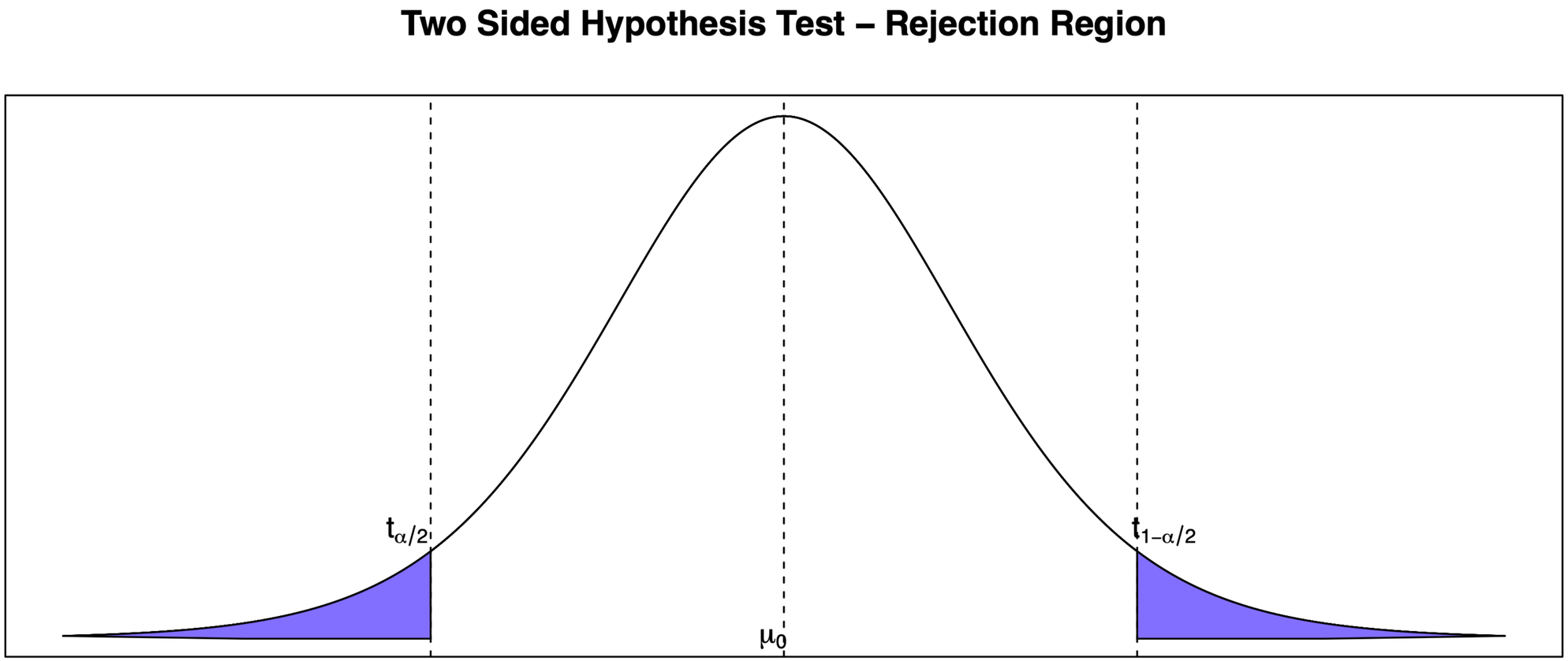


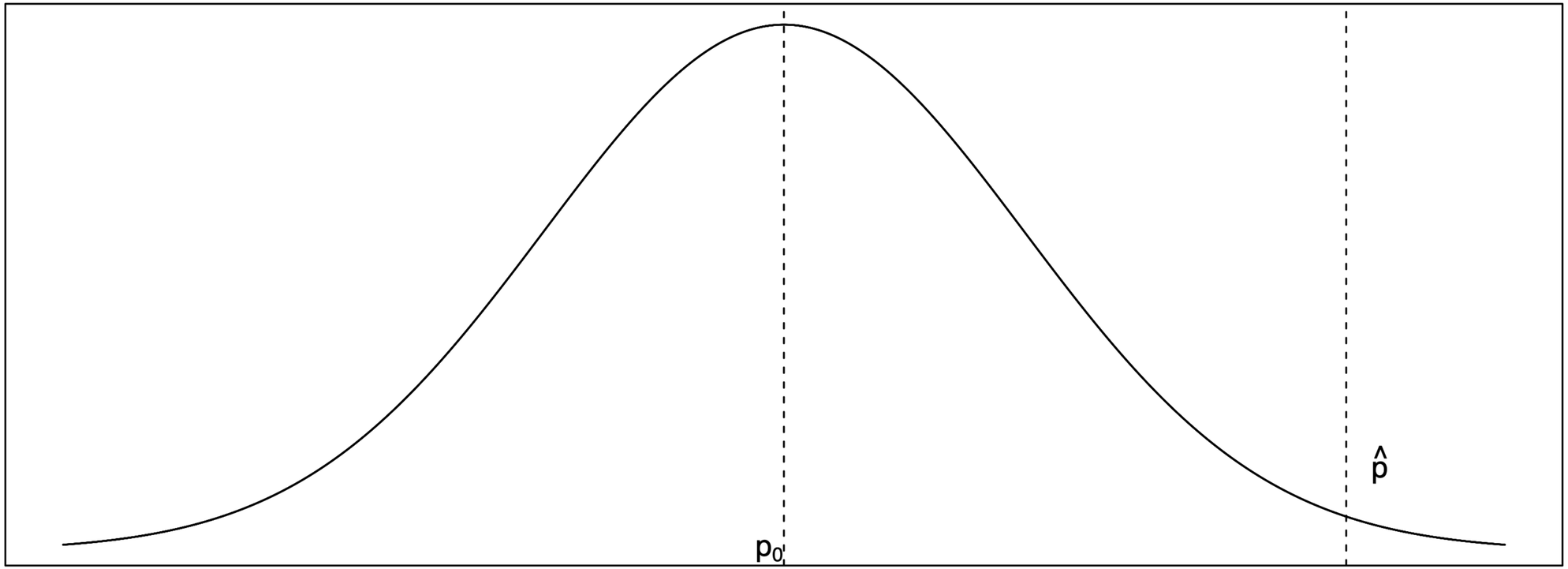
The Z Test for Population Proportions

- Suppose we have a sample from a binomial distribution with n trials and unknown p.
- We wish to test: $H_0 : p = p_0$ versus the alternative, $H_1 : p \neq p_0$.
  - May also consider $H_0 : p \leq p_0$ versus $H_1 : p > p_0$.
  - May also consider $H_0 : p \geq p_0$ versus $H_1 : p < p_0$.
- When the normal approximation applies we have seen that

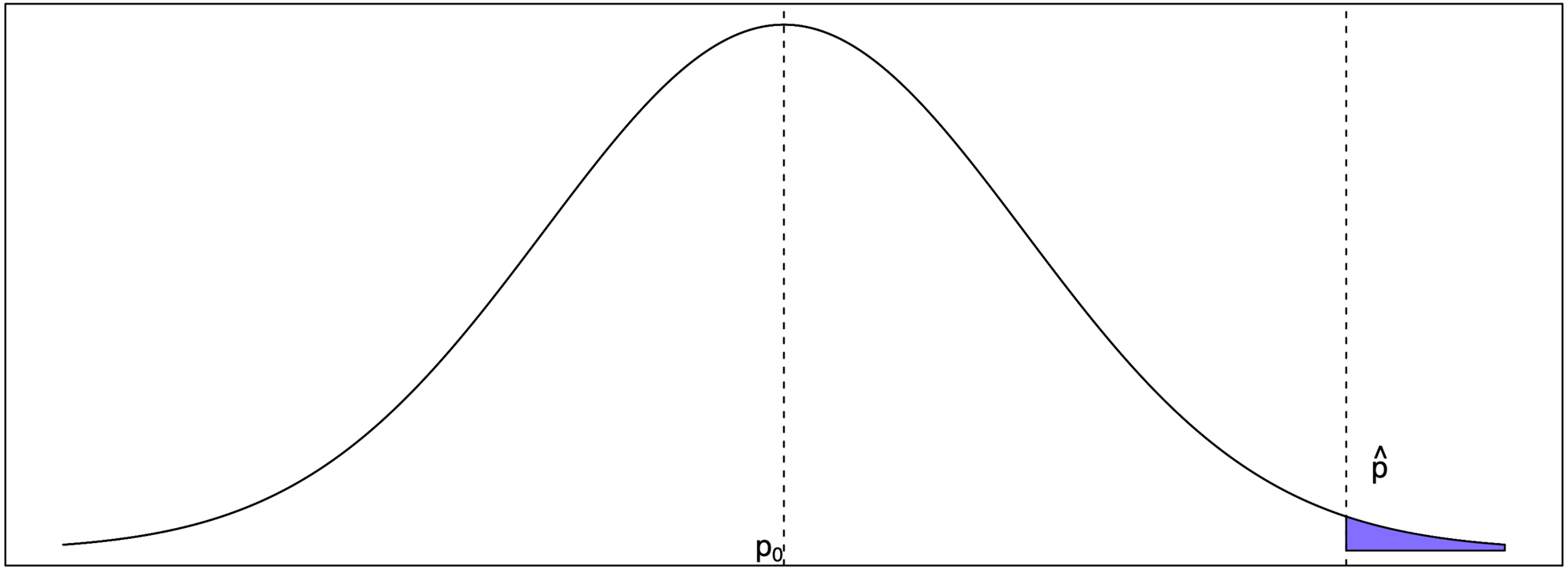
$$Z = \frac{\hat{p} - p_0}{\sqrt{np_0(1 - p_0)/n}}.$$

  - If p_0 is the correct mean, this will be approximately $N(0, 1)$.
- How do we find our p-value?

Calculating p-values



Calculating p-values



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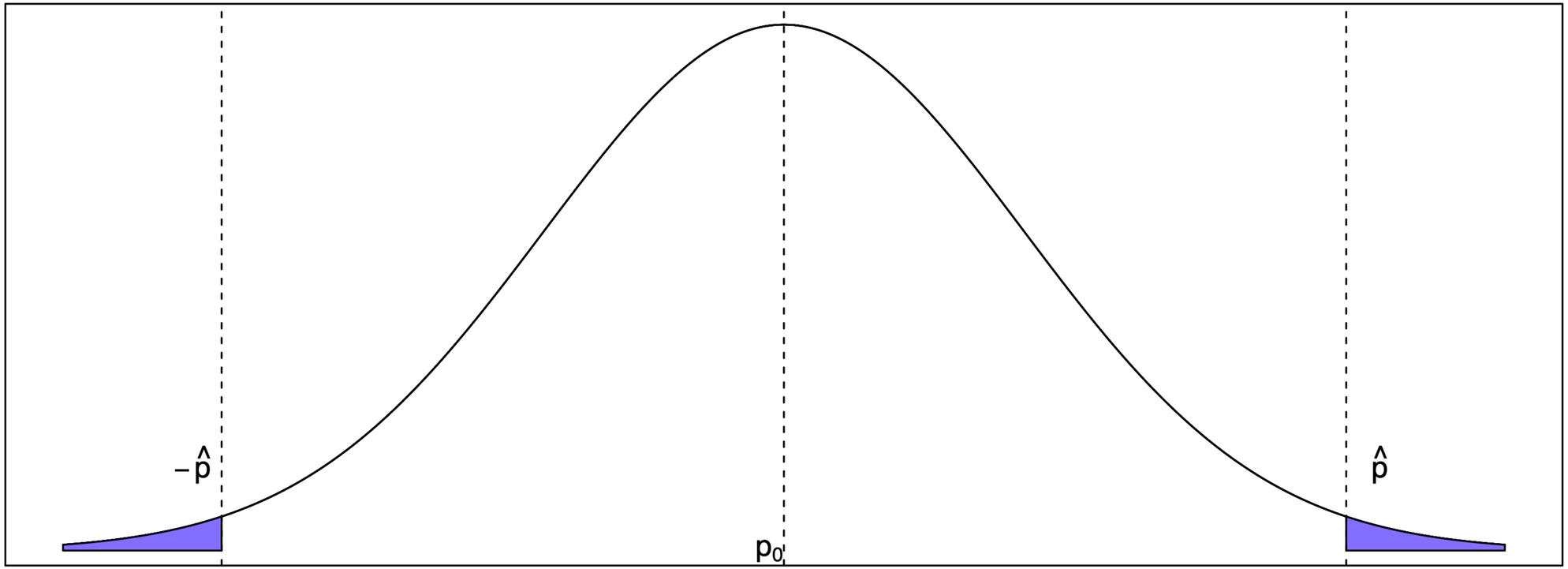

P-Value for (Two Sided) Tests

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Rejection Regions for Hypothesis Tests - Critical Values